# A Clarification on the Detection of Aharanov-Anandan$'$s phase and fault tolerate computation with symmetric SQUID

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**Abstract**

We point out that our scheme in manuscript quant-ph/0104127 is misunderstood by Alexander Blais and Andre-Marie S. Tremblay in their recent e-print quant-ph/0105006.

In our scheme(quant-ph/0104127), the state $|\pm>$(eigenstate of $\sigma_y$) will evolve cyclically, with a global $AA$ phase $\pm\gamma$. In the detection with single qubit, we assume the initial state of $|\psi_0>=|\downarrow>$. Note this initial state is a linear superposed state of $|\pm>$. Explicitly, $|\psi_0>=-\frac{i}{\sqrt{2}}(|+>-|->)$. Since we have known that in the evolution, state $|\pm>$ will change to $e^{\pm i\gamma}|\pm>$, the initial state $|\psi_0>$ will accordingly change into $|\psi(2\tau)>=-\frac{i}{\sqrt{2}}(e^{i\gamma}|+>-e^{-i\gamma}|->)=-\sin\gamma|\uparrow>-\cos\gamma|\downarrow>$. That is to say, *global* phase $\gamma$ to state $|\pm>$ is *not* a global phase to state $|\psi_0>$. Interference between $|\psi_0>$ and $|\psi(2\tau)$ *can* be observed. We have never claimed that we can observe the phase $\gamma$[1] through the interference between state $|+>$ and state $e^{i\gamma}|+>$. However, after obtaining the the pattern by $|<\psi_0|\psi(2\tau)>|^2$, one can deduce the value of $\gamma$, which is the AA phase shift of state $|+>$. Note $\gamma$ is not the AA phase shift of state $|\psi_0>$. This is to say, in order to detect the AA phase $\gamma$ to state $|+>$, we have to observe the interference pattern by initial state $|\psi_0>$ instead of $|+>$ itself.

To our understanding, the scheme in ref[2] to detect the Berry phase in the single qubit case also relies on the similar interference. There the initial state of qubit is not the eigenstate state of the initial Hamiltonian. The qubit will *not* undergo a closed path in the evolution. But the interference pattern is determined by the Berry phase shift to the eigenstate of Hamiltonian. Note what is detected there is the Berry phase shift to eigenstate of the Hamiltonian rather than the qubit state itself.

In the conditional Berry phase part in ref[2], the details are not given there. But comparing ref[2] with ref[3], we believe they have first adiabatically set the qubit to the eigenstate of the Hamiltonian and then operate it. This is different to the scheme described in the single qubit part in the same article. But in general it is not necessary to set the initial state of the qubit to the eigenstate of the Hamiltonian. Note the final goal is to create the conditional unitary transformation where no dynamic phase is involved.

Their note[1] on the fault tolerance part is interesting. But we can consider a type of operational error which causes the random fluctuation on the *state evolution path of* $|\pm>$. In this case, if the area enclosed by the initial state $|\pm>$ is preserved, the finally result is not distorted. Our scheme is fault tolerate to *certain* errors in the Hamiltonian provided the area enclosed by the state evolution path of $|\pm>$ is preserved under the those errors.

We believe our scheme for the single qubit case can be realized in laboratory immediately. We don′t know whether our scheme on conditional AA phase shift part can be realized in laboratory right now, because it is at least as difficult as the realization of normal C-NOT gate. But we believe that our scheme is (theoretically) much closer to the practical use than the old ones.

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**Reference**

[1] Alexandre B and Andre-Marie S M, quant/ph-0105006.
[2] C. Falci *et al*, Nature **407**, 355(2000).
[3] Jones *et al*, Nature **403**, 869(2000).